

CSC 314: Formal Languages

Exam 2, Second semester 2014/2015, Form: A

Name: _____

Student Number: _____

Section: _____

29.5
35

Section 1. (1 point each)

Mark the following statements with **True** if they are true and **False** otherwise.

- F A grammar $G = (V, T, S, P)$ is context-free if all productions have the form $A \rightarrow x$ where $A \in V$ and $x \in T^*$.
- F A grammar is ambiguous if you can derive two different words from its productions using a derivation tree or a left-most derivation or a right-most derivation.
- T Brute force can be used for parsing and answering the membership question if we eliminate unit and λ productions from the grammar.
- 8 T Every NPDA has a stack, and words are only accepted when the NPDA moves into a final state and the stack is empty.
- T The grammar $S \rightarrow aSb|ab$ is equivalent to $S \rightarrow aAb, A \rightarrow aAb|\lambda$.
- 1 T The intersection of two context free grammars is always context free.
- T The language $L = \{ww : w \in \{0, 1\}^+\}$ can be accepted by a Turing machine.
- T Any context free grammar can be converted to an NPDA that accepts the same language.
- T The grammar $S \rightarrow aSbbb|b$ generates the language $L = \{a^n b^{3n+1} : n \geq 0\}$.
- F The language $L(a^* b^* a a (a+b)^*)$ can be accepted by a Turing machine.

Section 2. (5 points each)

1. Consider the following language

$$L = \{a^n b^\ell : 3n \leq \ell \leq 5n, n \geq 0\}.$$

Prove that this language is context-free.

$$3n \leq \ell \leq 5n$$

$$3(n) \leq \ell \leq 5(n)$$

5

$$S \rightarrow aSB | \lambda$$

$$B \rightarrow bbb | bbbbb | bbbbbb$$



2. Show that the following grammar is ambiguous.

$$S \rightarrow ASB|ab|SS$$

$$A \rightarrow aA|\lambda$$

$$B \rightarrow bB|\lambda$$

$$SS \Rightarrow abS \Rightarrow \boxed{abab}$$

$$SS \Rightarrow ASBS \Rightarrow SBS \Rightarrow abBS \Rightarrow abS \Rightarrow \boxed{abab}$$

(5)

∴ Same word but different paths

∴ ambiguous

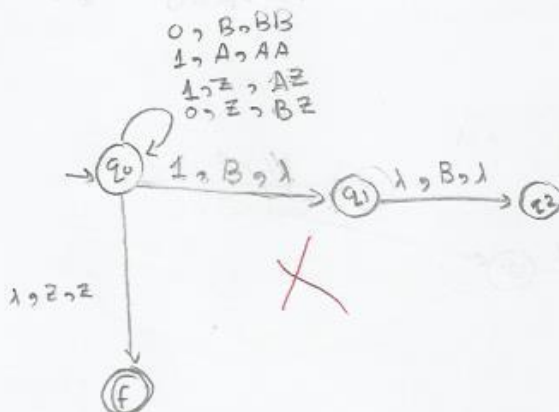
leftmost?

3. A signal is received in a remote weather station. The signal contains a sequence of n bits. In order to check if the signal contains errors the sending station encodes it so that the number of 0's is always twice the number of 1's.

Construct an NPDA that will check whether a given sequence of bits is acceptable or not according to this error checking scheme. Show your work and list your assumptions, if any.

(1.5)

1 circle 20



4. Convert the following context-free grammar to an equivalent NPDA that accepts the same language.

$S \rightarrow aaAB|bSb$
 $A \rightarrow bA|C$
 $B \rightarrow CBa|a$
 $C \rightarrow ab|b$

$b \rightarrow D, \lambda$
 $b \rightarrow C, \lambda$
 $a \rightarrow F, \lambda$
 $a \rightarrow C, D$
 $a \rightarrow B, \lambda$
 $b \rightarrow B, BF$
 $a \rightarrow B, DBF$
 $b \rightarrow A, \lambda$
 $a \rightarrow A, D$
 $b \rightarrow A, A$
 $b \rightarrow S, SD$
 $a \rightarrow S, FAB$

5

$S \rightarrow aFAB|bSD$

$A \rightarrow bA|aD|b$

$B \rightarrow aBa|bBa|a$
 $B \rightarrow aDBF|bBF|a$

$C \rightarrow aD|b$

$D \rightarrow b$

$F \rightarrow a$



5. Construct a Turing machine that accepts the following language.

$L = \{a^n b^{n+m} c^m : n, m \geq 0\}$

5

$a a b b b b c c$
 $x x y y y y z z$

